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A robust method for investigating galactic evolution in the submillimetre waveband: II the submillimetre background and source counts

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Abstract

This is the second of two papers describing a model of galactic evolution in the submillimetre waveband. The model incorporates a self-consistent treatment of the evolution of dust and stars, is normalized to the submillimetre properties of galaxies in the local universe, and can be used to make predictions for both disk and elliptical galaxies and for ‘closed-box’, ‘inflow’, and ‘outflow’ models of galactic evolution. The model does not include the effects of hierarchical clustering, but we show that the variation in the predictions produced by the different dust-evolution models is so large that it is premature to include the effects of an even more uncertain process. In Paper I we investigated whether it is possible to explain the extreme dust masses of high-redshift quasars and radio galaxies by galactic evolution. In this paper we use the model to make predictions of the submillimetre background and source counts.

All our disk-galaxy models exceed at short wavelengths ($\lambda < 200\mu\text{m}$) the submillimetre background recently measured by Puget et al. (1996), suggesting that either there is a problem with our models, or that the background measurement at the shorter wavelengths is inaccurate, or a combination of the two. However, the two models in which we assume the rapidly-evolving star-formation rate found from optical studies predict backgrounds that are so much greater than the measured background that we do not believe that this discrepancy can be due to an inaccurate background measurement. We therefore conclude that there is no evidence in the submillimetre waveband for the rapid evolution found from optical studies. At longer wavelengths, most of the disk-galaxy models predict backgrounds that are less than the observed background. Our elliptical-galaxy models are more uncertain because they are less securely tied to the observed submillimetre properties of the local universe. We find that it is relatively easy to produce models that predict backgrounds similar to that observed, but we caution that our disk-galaxy models show that a significant fraction of the observed background must be coming from disk galaxies. The biggest weaknesses in our models are the lack of a direct measurement of the submillimetre luminosity function for galaxies and the fact that we have been forced to assume that all local disk-galaxies have the same far-infrared—submillimetre spectral energy distribution and the same ratio of gas mass to stellar mass. Observations with the SCUBA submillimetre array should remove both of these weaknesses, as well as providing measured source counts with which to confront the models.

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Introduction

The submillimetre waveband ($100\ \mu\text{m} < \lambda < 1\ \text{mm}$) is one of the few electromagnetic wavebands that has not been used extensively for astronomical observations. In particular, our knowledge of the submillimetre properties of galaxies is very poor, with there only being a handful of submillimetre flux measurements of galaxies (Eales, Wynn-Williams & Duncan 1989; Stark et al. 1989). Our ignorance is due partly to there only being a few atmospheric windows within this waveband and partly to technological limitations: continuum observations in this waveband have relied on single bolometers, which means that only a single point in the sky can be observed at one time. However, the situation is rapidly changing. As we write, the Infrared Space Observatory (ISO) is making observations at wavelengths up to $200\ \mu\text{m}$, thus covering the short-wavelength end of the submillimetre waveband, and an array of bolometers (the **S**ubmillimetre **C**ommon **U**ser **B**olometer **A**rray—SCUBA [Duncan 1990]) is being installed on the James Clerk Maxwell Telescope. The advantage of the latter instrument is that, first, of course, arrays make it possible to map the sky much more quickly but, second, and perhaps more important, the individual bolometers in SCUBA are ten times more sensitive than previous bolometers. Finally, there is an instrument that is no longer operating: the Cosmic Background Explorer Satellite. COBE was designed to measure the integrated infrared and submillimetre emission from galaxies, and recently Puget et al. (1996) have claimed to find, in the vast COBE database, evidence for this emission.

The submillimetre waveband is important to our understanding of galaxies for two reasons. The most basic reason is that galaxies do emit a significant fraction of their emission in this waveband (Eales, Wynn-Williams & Duncan 1989; Stark et al. 1989), so a proper energy budget for individual galaxies is impossible without submillimetre observations. The second more important reason is that the presence of dust means our view of the galaxy population as a whole is always biased at optical wavelengths—a bias which can be corrected by observing the submillimetre emission from the dust that is doing the obscuring. An example of this is the question of the origin of elliptical galaxies. Since there are large numbers of old stars in these objects but very little current star formation, it seems likely that the stars were formed over a relatively short period. However, searches at optical wavelengths have failed to find any evidence of a cosmic epoch in which this star formation is occurring (e.g. De Propris et al. 1993). A possible solution to this problem is if the galaxies in this phase are shrouded with dust: in which case the galaxies will be submillimetre sources (Bond, Carr & Hogan 1986).

There has been a large number of attempts to predict the submillimetre background and source counts that should be produced by the ‘universe of galaxies’. Accurate predictions require knowledge of two things which are not well known: the submillimetre properties of the local universe and how galaxies evolve. Previous models of the background and the source counts generally fall into two groups. In the first fall those which are tied closely to observations but make no attempt to incorporate a physical model for how galaxies evolve. For example, Beichman & Helou (1991) and Eales (1991) based their models on the local far-infrared luminosity function and on observational evidence about the spectral energy distributions of galaxies in the far-infrared–submillimetre waveband but made no attempt to incorporate, our admittedly limited, understanding of how galaxies

evolve; instead they just assumed simple functional forms for the evolution and investigated how the backgrounds and the counts changed as the parameters of these functions were changed. Our criticism of these models is that we do know *something* about the physics of galaxy evolution and this should be incorporated in the models. In the other camp are the models which incorporate detailed physical models for how galaxies evolve (Wang 1991a,b; Franceschini et al. 1994). Our minor criticism of these models is that the treatment of what is known about the submillimetre properties of galaxies tends to be rather cursory. Our major criticism is that they tend to be very complex with large numbers of parameters, which creates a number of related problems. The complexity means that it is usually unclear how critically the predictions depend on the accuracy of the assumptions of the model. Wang's models (Wang 1991a), for example, contain assumptions about how dust is created and destroyed in the interstellar medium, processes which are still poorly understood (Whittet 1991), and it is unclear how critically the model predictions depend on these assumptions. From a practical observer's point of view, one would like to be able to see which are the critical assumptions in any model and whether these are susceptible to observational confirmation.

Encouraged by the new observational opportunities, we have started a project to model galaxy evolution in the submillimetre waveband. Our project has three goals. We want our models to incorporate all that is known about galaxies in this waveband and, as the observations improve, we want it to be easy to include the new information. We want to incorporate a physical model for how galaxies evolve, but one which, while covering as wide a variety of types of evolution as possible, is also fairly simple. Lastly, we want to be able to determine the critical assumptions on which the model is based and whether these can be improved by future observational or theoretical work. Given the new submillimetre instruments, which will make all kinds of new observational projects possible, this final point is an important one. As an example, we find that, as expected, the strength of the submillimetre background can be used to distinguish between types of galactic evolution, but that our predictions depend critically on our assumptions about the submillimetre properties of nearby galaxies: and thus, something which is not obvious, a key cosmological project to carry with the new submillimetre arrays is actually to observe large numbers of nearby galaxies.

There are two groups who have carried out projects with similar objectives to ours. Blain & Longair (1993, 1996) have investigated the effect of the evolution of large-scale structure on the submillimetre background and source counts. We decided not to include the effects of this in our models, partly because there is no consensus about any aspect of the evolution of large-scale structure, but mainly because we find the uncertainties in the evolution of dust in individual galaxies (something not considered by Blain & Longair) lead to a large spread in the predicted background and counts, without including the much more uncertain process of large-scale structure evolution. Fall, Charlot & Pei (1996) have recently presented models of the evolution of Ly α absorption systems which are an elegant synthesis of our current ideas about galactic evolution and of what is known about observationally about these objects. These models produce estimates of the backgrounds in all wavebands. Our approach is quite different from that of Fall et al., since we discard generality and elegance, and aim to produce a very simple model, as closely linked to the

present submillimetre observations as possible, and one possible to easily modify when new submillimetre data arrives.

In the first paper (Eales & Edmunds 1996; henceforth EE), we used our model to show that galactic evolution can account for the high dust masses of a handful of high-redshift quasars and galaxies but only for a very restricted range of model parameters. In this paper we use our models to make predictions for the submillimetre background and source counts. The details of the models are in §2. The results and discussion are in §4. Since we do not want to interrupt the flow of the paper to analyse all the uncertainties in the models, we have noted every place where we think there is some uncertainty by a ‘U’ in parentheses, with usually a short discussion in a footnote. Finally, we note that as one of our goals is to give observers a relatively straightforward theoretical framework with which to start to interpret the avalanche of submillimetre observations of galaxies that is (hopefully) about to occur, copies of the software used in this paper, which can be used to generate backgrounds and source counts at any wavelength can be obtained from the authors.

2. The Models

2.1 Models of how the dust mass evolves

Our models are based on the fundamental assumption that the mass of dust in a galaxy is always proportional to the mass of metals in the interstellar medium. The observational evidence that this is true at both high and low redshift is fairly good and is given in EE (U†). The advantage of this assumption is that we can sidestep the many uncertainties about the properties of dust. A second less fundamental assumption, which we make for convenience, is that there is no significant delay between the formation of a coeval population of stars and the production of dust by this population (see EE for a discussion).

Once we have made these assumptions, we can use standard chemical evolution models, for which we use the notation of Edmunds (1990). We assume (like Fall, Charlot & Pei [1996]) that the dust mass M_d present in a galaxy is simply proportional to the product zg of the mass fraction z of heavy elements in the interstellar medium with the gas mass g . We allow inflow of unenriched gas into the system or outflow of gas at the ambient metallicity, and use the simplest ‘linear’ models (e.g. Edmunds 1990, and references therein) in which the inflow or outflow rate is linearly proportional to the star-formation rate (SFR: inflow = $\gamma \times \text{SFR}$ or outflow = $\lambda \times \text{SFR}$). Although real galaxies will have more complicated variations of inflow or outflow, these models should give a reasonable idea of the likely effects of different types of chemical evolution, and have the advantage that the time variation of the star formation need not be made explicit. The total mass

† Our models do not rely on us knowing the absolute value of the fraction of metals bound up in dust, merely that this ratio does not change with redshift. If this ratio is different by, for example, a factor of two at high redshift than it is at low redshift, our predicted submillimetre fluxes for the high-redshift objects will be off by the same factor. For further discussion, see EE.

M_{tot} of the galaxy at any time is the sum of the gas mass g (which includes the dust mass) and the stellar mass αs , s being the total mass of stars that have formed and α being the mass fraction of the stars which remains locked up in long-lived low-mass stars or stellar remnants (we assume a value for α of 0.8). The gas fraction f is defined as gas mass/total mass. It can be shown that, for the three different scenarios of galactic evolution—closed-box, inflow, and outflow—and starting from an initial unit mass of gas, the following equations hold:

Closed-box:

$$M_d = k f \ln(1/f),$$

in which k is the product of the yield (Edmunds 1990) and the fraction by mass of the metals in the interstellar medium that are incorporated in dust.

$$\alpha s = 1 - f,$$

$$M_{tot} = 1.$$

Outflow:

$$M_d = \frac{k g \ln(1/g)}{1 + \lambda/\alpha},$$

$$g = \frac{f}{1 + (\lambda/\alpha)(1 - f)},$$

$$\alpha s = \frac{1 - f}{1 + (\lambda/\alpha)(1 - f)},$$

$$M_{tot} = \frac{1 + (\lambda/\alpha)g}{1 + (\lambda/\alpha)}.$$

Inflow:

$$M_d = \frac{k g}{\gamma/\alpha} (1 - g^{(\gamma/\alpha)/(1-\gamma/\alpha)}),$$

$$g = \frac{f}{1 - \gamma/\alpha(1 - f)},$$

$$\alpha s = \frac{1 - f}{1 - (\gamma/\alpha)(1 - f)},$$

$$M_{tot} = \frac{1 - (\gamma/\alpha)g}{(1 - (\gamma/\alpha))}.$$

A way of getting insight into these equations is to normalize them to the properties of low-redshift galaxies. Low-redshift spirals typically have values of f of about 0.1 (Young & Scoville 1991). Figure 1 shows the ratio of the dust mass at a gas fraction f to the dust mass at a gas fraction $f = 0.1$ i.e. the current value. When a galaxy is formed it contains no stars and so $f = 1$. When star formation starts, metals, and thus dust, start to be created, leading to a gradual increase in the total dust mass. However, star formation also consumes gas and the dust which is assumed to be intermingled with the gas. Eventually this second process becomes the most important one, leading to a decrease in the dust mass. Including inflow or outflow in the model changes the relative efficiencies of the two processes. The rather surprising conclusion from these curves is that the dust masses of spiral galaxies are unlikely to have been significantly higher in the past than they are today. Only the outflow models produce dust masses in the past substantially higher than those today, and the largest increase in dust mass in Fig. 1 is probably unrealistic because, as we argue in EE, a value for λ/α of 10.0 leads to too low metal abundances for present-day galaxies. Thus the largest increase in the dust mass that is possible, even when outflow models are considered, is $\simeq 4$.

Since cosmic time does not come into these equations, these models can be applied to ellipticals as well as to spirals, although as the current value for f of ellipticals is much less than 0.1, the curves in Fig. 1 are not particularly useful. Once a star-formation history is assumed for a galaxy, the equations can be used to calculate how dust mass depends on cosmic time. If the bulk of the star formation in an elliptical galaxy does occur over a short period (§1), the evolution of the dust mass seen in Fig. 1 will occur but over this short period, with the maximum dust mass being the same as the maximum dust mass of a spiral of the same total mass.

2.2 Extending the models to submillimetre luminosity

Once a history of star formation is assumed for a galaxy the change of the dust mass with time can be calculated using the equations above. However, this is not enough to predict how the submillimetre luminosity depends on time. To predict this, we have to make answer two questions: (1) What are the stars that are heating the dust? (2) Is the dust in a galaxy absorbing most of the optical and ultraviolet light from the stars (the dust is optically-thick) or is only a small fraction of this light being absorbed (the dust is optically-thin)?

There is a long-running argument in the literature as to whether the bulk of the far-infrared emission from galaxies is from dust heated by old low-mass stars or from high-mass stars (e.g. Helou 1986, Persson & Helou 1987; Boulanger & Perault 1988; Devereux & Young 1990) which seems to have petered out through exhaustion rather than consensus.

We shall assume that the far-infrared and submillimetre emission is from dust that is heated by young massive stars. The reason for this assumption is the practical one that we can then link the far-infrared-submillimetre properties of a galaxy directly to the current star-formation rate in the galaxy. This assumption may not be correct in present-day spirals with relatively low star-formation rates, but for the majority of the models in which we assume that the star-formation rate is constant with time this actually doesn't matter (see below); and for the models in which the star-formation rate was higher in the past, even if the assumption is not completely correct at low redshift, it will be increasingly good as one moves to a higher redshift.

For a typical starburst galaxy, a much larger fraction of the bolometric luminosity is coming out in the far-infrared—submillimetre wavebands than in the optical waveband (Soifer, Houck & Neugebauer 1987), and therefore the dust in the galaxy must be largely optically-thick to the starlight. However, in galaxies in the standard optical catalogues, the ratio of far-infrared luminosity to optical luminosity is often much less than one, suggesting that the dust is largely optically thin to starlight (Soifer, Houck & Neugebauer 1987). It is, of course, possible that even if much of the optical light is escaping from a galaxy, the ultraviolet light, what the massive young stars mainly produce, may be largely being absorbed by the dust, but, as far as we know, the quantitative far-infrared—ultraviolet comparison of a large sample of galaxies which is needed to answer this point has not been done. The necessity of deciding whether galaxies are optically-thin or optically-thick is because this determines how one predicts the evolution of the submillimetre luminosity of a galaxy. If a galaxy is optically-thick, the submillimetre luminosity is just proportional to the star-formation rate in the galaxy, and the evolution of the dust mass, and the elaborate apparatus we constructed §2.1, is irrelevant. If a galaxy is optically-thin, its submillimetre luminosity depends on both the dust mass and the dust temperature, which we can calculate from the star-formation rate. In reality, of course, a given galaxy may pass through optically-thin and optically-thick stages. This becomes too complex to model, and instead we have constructed some models in which all the galaxies are assumed to be optically-thin and some in which all the galaxies are assumed to be optically-thick.

2.2.1. The optically-thin models

If a galaxy is optically-thin to starlight, its submillimetre luminosity, L_ν , is given by

$$L_\nu = M_d \kappa_d(\nu) B(\nu, T_d),$$

in which M_d is dust mass, $\kappa_d(\nu)$ is the dust absorption per unit mass, and $B(\nu, T_d)$ is the Planck function. The absolute value of $\kappa_d(\nu)$ is poorly known (Hughes et al. 1993) but its frequency dependence is believed to lie between $\kappa_d \propto \nu$ and $\kappa_d \propto \nu^2$ (Whittet 1991). The temperature of the dust, T_d , depends on the intensity of the interstellar radiation field, and, if, as we assume, this is dominated by high-mass stars, then $T_d \propto (\text{star formation rate})^{1/(4+n)}$, where n is the index in the relation $\kappa_d \propto \nu^n$. Thus, if we know the history of star formation in a galaxy, we can calculate how both the dust mass and the dust temperature depend on time. However, since we do not know the constant

of proportionality in the relation between dust temperature and star-formation rate and since the absolute value of the dust absorption coefficient is poorly known (e.g. Hughes et al. 1993), we cannot immediately make an accurate prediction of how the submillimetre luminosity depends on time. To do this, we have to normalize the models to the *observed* submillimetre properties of nearby galaxies.

Suppose there is a galaxy for which we know the far-infrared—submillimetre spectral energy distribution and the relative amounts of gas and stars. We can estimate the dust temperature from the spectral energy distribution, and if we make an assumption about the past star-formation rate relative to the current star-formation rate, we can calculate how the dust temperature has changed with time. Since we know the relative amounts of gas and stars, we know the current value of the f parameter (§2.1) and hence we can calculate the evolution of the dust mass as a function of the current dust mass. With this information, and since we know the current submillimetre luminosity, we can calculate the evolution of the submillimetre luminosity for this galaxy. Note that following this procedure means that we have avoided the difficulty of not having an accurate value for the dust absorption coefficient, and we also do not need to know the current value of the star-formation rate in the galaxy, something which observationally is very difficult to measure.

2.2.2. The optically-thick models

If a galaxy is optically-thick, the thermal emission from the dust in the galaxy is simply proportional to the star-formation rate and is independent of the evolution of the dust mass. Thus, if one knows the current submillimetre luminosity of a galaxy and one makes an assumption about its star-formation history, it is a simple matter to calculate how its submillimetre luminosity should change with time. This skates over one difficulty, because it is possible that the dust temperature will also change, which, while keeping the total thermal emission the same, will change the spectral energy distribution of the dust. However, whether or not the dust temperature changes will depend on what assumptions one makes about the distribution of the star formation, and the simplest first assumption to make is that the dust temperature doesn't change.

2.3. Statistical Predictions

Once one has made assumptions about whether galaxies are optically-thin or optically-thick and about the star-formation histories of galaxies, it is straightforward to predict the submillimetre properties of the high-redshift universe, as long as we know the submillimetre luminosities of all the galaxies in the local universe (optically-thick case) or the spectral energy distributions and the ratios of stellar to gas mass for all the galaxies in the local universe (optically-thin case). Of course we do not have this information, so we have to resort to piecing together statistically the limited amount of information that we do have.

The local submillimetre luminosity function for galaxies is not known directly but, as a result of the IRAS survey, the luminosity function at $60\mu\text{m}$ is known very well (Lawrence et al. 1986). With some assumption about the average far-infrared—submillimetre spectral energy distribution (SED) of galaxies it is therefore possible to use the $60\mu\text{m}$ luminosity

function to make an estimate of the submillimetre luminosity function. The drawback of making predictions based on this estimated local submillimetre luminosity function is that elliptical galaxies currently contain very little dust and are therefore very weak sources of far-infrared emission (Fich & Hodge 1993), and are so not adequately represented in the $60\mu\text{m}$ luminosity function. For this reason we have divided the galaxy population into two classes: those well represented in the $60\mu\text{m}$ luminosity function, a mixture of spirals and irregulars, which we will broadly label as “disk systems”, and elliptical galaxies.

2.3.1. Disk Galaxies

Our method of predicting the contribution of disk galaxies to the background and to the source counts is quite straightforward. We start with the $60\mu\text{m}$ luminosity function of Lawrence et al. (1986). Given the lack of submillimetre measurements of galaxies (§1), it is impossible to do anything sophisticated such as investigating whether a galaxy’s SED in the far-infrared and submillimetre wavebands is a function of morphology or far-infrared luminosity. Instead, we have taken all the available data, both observations of other galaxies and COBE results for our own galaxy, and shown that all the observations are broadly consistent with a single SED for disk galaxies (Fig. 2). This SED can be represented as two grey-bodies with temperatures of 27K and 150K, with the ratio of the masses of dust in the two components being $10^4:1$. For full details see the caption to Fig. 2.

In the optically-thick case it is straightforward to use the standard cosmological formulae, this SED, and the $60\mu\text{m}$ luminosity function to predict the submillimetre background and source counts. We have constructed two basic models. In the first, we just assume that the star-formation rate does not change with time, something which is suggested by optical observations of nearby spiral galaxies (Kennicutt 1983). In this case the submillimetre luminosity of a galaxy will also be a constant. In the second case we used the star-formation history inferred by Pei and Fall (1995) from a study of the statistics of Ly α absorption systems. In this the star-formation rate increases rapidly from the current epoch to a redshift of one, falling off thereafter; the initial rapid rise in star-formation rate being similar to that inferred from the Canada-France Redshift Survey (Lilly et al. 1996). As we discussed above, in the optically-thick approximation the submillimetre luminosity simply scales as this star-formation rate. In both models we only considered galaxies out to a maximum redshift (z_{max}), equivalent to assuming that the galaxies form at this redshift. The predicted backgrounds for $z_{\text{max}} = 3$ are shown in Figure 3 and those for $z_{\text{max}} = 1$ in Figure 4. The background that Puget et al. (1996) have claimed to find in the COBE database is shown in both figures, as are the upper limits on the background that Davies et al. (1996) have derived from the DIRBE experiment on COBE. Figure 6 shows the predicted source counts at $190\mu\text{m}$, one of the main ISO wavelengths, and Figure 7 shows the predicted source counts at $850\mu\text{m}$, the main SCUBA wavelength.

In the optically-thin case, we have to make an additional assumption about the ratio of gas mass to stellar mass in present-day galaxies. Following the results of Young & Scoville (1991), we assume that all present-day disk galaxies have a ratio of gas mass to stellar

mass of 1:9 (U^\dagger). We tried the same two star-formation histories as in the optically-thin case and used a number of different models for the evolution of the dust mass. We also tried the same two values for z_{max} as were used in the optically-thick case. The predicted backgrounds are also shown in Figs 3 & 4, where we have tried to show enough models that the effect of adjusting the different input parameters (Ω_0 , changing from inflow to outflow, changing the dust parameter in an outflow model, to give three examples) is clear. Figures 6 & 7 show the predicted source counts.

2.3.2. Elliptical Galaxies

Our method of predicting the contribution of elliptical galaxies to the background and the source counts is more complicated and, in order not to complicate the method even more, we just consider optically-thin models. We consider the qualitative effect of changing from an optically-thin model to an optically-thick model at the end of this section.

We assume that star formation in a particular galaxy starts at some redshift, continues at a constant rate for a fixed time, and then stops, all the gas having been used up—a process we loosely refer to as ‘galaxy formation’. The parameters that have to be put into the model are the range of redshifts over which galaxy formation occurs and the time it takes an individual galaxy to form. If this time, τ , is less than the duration of the galaxy-formation epoch, τ_{gf} , we assume that the fraction of galaxies that are forming at any one time during this epoch is τ/τ_{gf} .

Before we continue with the details of the model it is instructive to consider some of its broad implications. During the formation period the dust mass in the galaxy follows one of the curves shown in Fig. 1, ending at zero as we are assuming that the formation period ends when all the gas has been turned into stars. Changing the value of τ will not change the maximum dust mass, it will merely cause the dust mass to continue along its evolutionary track either more or less quickly. However, the submillimetre luminosity of the galaxy will change because the stars are forming at a different rate, and thus the interstellar radiation field and hence the dust temperature will be different. Changing the value of τ from 1 Gyr to 0.1 Gyr, for example, will increase the star formation rate by a factor of 10, and hence the dust temperature by a factor of $10^{1/(4+n)} \sim 1.5$. The effect of this rise in dust temperature on the predicted submillimetre flux will depend greatly on the wavelength of the observation. If one is observing on the Rayleigh-Jeans side of the thermal peak, the increase in submillimetre flux will be only this same modest factor, whereas if one is observing on the Wien side the increase can be very large indeed. This is an example of the general point that observations on the Rayleigh-Jeans side of the thermal peak are primarily sensitive to dust mass, whereas those on the Wien side are primarily sensitive to dust temperature. This argument can also be used to place an approximate upper limit on the dust temperature of an elliptical galaxy. Disk galaxies typically have dust temperatures of 20-30K (Eales, Wynn-Williams & Duncan 1989; Stark et al. 1989). If we assume that

[†] This is undoubtedly simplistic, because Young and Scoville’s result show that this ratio is a function of Hubble type, but it is the best we can do at the moment. If present-day disk galaxies generally have a smaller ratio of gas to dust, the submillimetre luminosities at high redshift will be increased (Fig. 1), and vice versa.

these have been forming stars at roughly the same rate for the age of the universe, and if we take the free-fall time ($\simeq 0.1$ Gyr—Fall & Rees 1985) as the smallest value for the formation period of elliptical galaxies, the maximum dust temperature for elliptical galaxies with the same mass as low-redshift spirals is $\sim 25 \times 1000^{1/(4+n)} \simeq 88\text{K}$, consistent with the dust temperatures that have been measured for two high-redshift objects (Downes et al. 1992; Isaak et al. 1994).

The problem in implementing the models, as with the disk galaxies, is that we need some way of normalizing the models to the observed submillimetre properties of galaxies, in order to avoid the necessity of having to calculate absolute dust masses and of needing to know the constant of proportionality in the relation between dust temperature and star-formation rate. We can do this for elliptical galaxies, but in a less satisfactory way than for disk galaxies. We use the spiral galaxy NGC 4254, which was observed by Stark et al. (1989) and has a dust temperature (assuming $\kappa_d \propto \nu^2$) of 23K. This is not a special galaxy, and any galaxy would do for which it is possible to estimate a dust temperature. We assume that, as for most spiral galaxies (Young and Scoville 1991), the ratio of gas mass to total mass (gas and stars) is 0.1. Once we have done this, we can calculate the submillimetre luminosity of an elliptical galaxy by scaling from the properties of NGC 4254. The submillimetre luminosity of the elliptical galaxy is given by

$$L_\nu = L_{\nu, NGC4254} 10^{\frac{(M_{NGC4254} - M)}{2.5}} \frac{(M/L)_{elliptical}}{(M/L)_{disk}} \times 23 \left(\frac{\tau_h}{\tau}\right)^{1/(4+n)} \times \left(\frac{M_d(f)}{M_d(0.1)}\right),$$

in which $L_{\nu, NGC4254}$ is the submillimetre luminosity of NGC 4254, M is the absolute magnitude the elliptical galaxy has at the current epoch, $M_{NGC4254}$ is the absolute magnitude of NGC 4254, τ_h is the age of the universe, and M_d is the dust mass as a function of the parameter f , which measures how far the elliptical galaxy is through the formation process (§2.1). Using the results of Persic and Salucci (1992), we assume that the ratio of the baryonic mass-to-light of ellipticals to that of spirals is 3.6. We use a value for n of 2.

Figure 5 shows the predicted background for a number of models. We have again tried to show enough models that the effect of adjusting the different input parameters is clear. Figures 8 and 9 show the source counts predicted at 190 and 850 μm .

In order not to increase the complexity of the models, we have not constructed optically-thick models. It is clear, however, that the qualitative effect on the background predicted by a particular model of going from the optically-thin assumption to the optically-thick assumption would be to increase the brightness of the background, since none of the optical light would then be escaping, but to leave the spectral shape roughly the same, since this is largely dependent on the redshift at which the galaxies are assumed to form.

3. Discussion

One of the most obvious features of the disk-galaxy predictions is that at short wavelengths ($\lambda < 200\mu\text{m}$) all of the models predict a higher background than was actually

measured by Puget et al. (1996). There are two obvious possible explanations of this. First, Puget et al.’s estimate of the background may be too low, which is perfectly possible given the complicated technique they were forced to use to remove the foreground emission. Second, our models may be wrong in some way. This might be caused by a fundamental error in the input physics, or it might be caused by one of the simplifying assumptions we were forced to make because of our ignorance of the submillimetre properties of the nearby universe, on which the models rely. We can think of two main problems. First, we do not have a direct measure of the local submillimetre luminosity function and were forced to extrapolate from the far infrared. Second, we had to “lump” all disk galaxies together and assume a single far-infrared—submillimetre spectral energy distribution and a single value for the ratio of gas mass to stellar mass; and we already know this second assumption, at least, is wrong, because Young & Scoville (1991) have found that this ratio is a function of Hubble type. The advent of SCUBA should remove most of these problems, because with SCUBA it will be possible to measure the submillimetre luminosity function directly, and once submillimetre fluxes exist for large numbers of local galaxies it will be possible to predict the background produced by different groups of galaxies, divided either by Hubble type, far-infrared luminosity, or by some other criterion.

Although all the models predict too much background at the short wavelengths, there are two models which predict a background which is much greater than Puget et al.’s background estimate. These models also exceed the upper limit on the background estimated by Davies et al. (1996) from COBE data, and since these limits were produced in a relatively straightforward way, we feel that here the problem must clearly lie with the models. The two models that are very discrepant are the ones in which we assume the star-formation history proposed by Pei & Fall (1995) from a study of the statistics of quasar absorption lines, a history in which the star-formation rate at $z \sim 1$ is much greater than that today, something which has also been claimed on the basis of the results from the Canada-France Redshift Survey (Lilly et al. 1996). Since one of the two models is the optically-thin model and the other the optically-thick model, it seems clear that if the star-formation rate in galaxies is increasing rapidly with redshift, we are not seeing any evidence for this in the submillimetre waveband.

At longer wavelengths ($\lambda > 350 \mu m$), the disk-galaxy predictions are generally lower than the observed background, but still make a significant contribution to the background.

The elliptical galaxy models lead to a wide variety of predictions, and it is clear from an inspection of Fig. 5 that a fine-tuning of the parameters would lead to a model that would fit the background rather well. The model that produces a background with a shape most similar to that of the observed background is a closed-box model with galaxies forming during the range of redshifts $2 < z < 5$, with a formation period of 1 Gyr. The predicted background is about a factor of two lower than the observed background, but this is not a large factor given the extra uncertainties that went into the elliptical models (§2.3.2). Nevertheless, we caution that simply trying to match the elliptical models to the observed background is not appropriate, since the disk-galaxy models show that a significant fraction of the background must be produced by galactic disks.

One surprising feature of our results is that the disk galaxies appear to be producing a stronger background than the elliptical galaxies, which is in conflict with arguments

based on global metallicity. The integrated background light produced by a population of objects at a redshift z is related to the smoothed-out cosmic density of processed material, $\langle \rho(Z + \Delta Y) \rangle$, produced by those objects by

$$\int_0^\infty I_\nu d\nu = \frac{0.007 \langle \rho(Z + \Delta Y) \rangle c^3}{4\pi(1+z)}$$

(e.g. Pagel 1993). Pagel claims that the smoothed-out density of metals produced by elliptical systems is $\simeq 6$ times greater than that produced by spirals, which should lead, using this equation and ignoring for the moment the redshift factor, to a background from the elliptical galaxies $\simeq 6$ times higher than that from the spirals. We find rather the reverse, with there being typically (the exact difference depends on which models are being compared) a factor of five the other way. We can account for part of this difference with the redshift factor. In our elliptical models the galaxies are at much higher redshifts than in the disk models. However, this usually only amounts to a factor of $\sim 3 - 4$, still leaving a factor of ~ 10 discrepancy.

The only explanation we can think of for this discrepancy is if much of the far-infrared—submillimetre emission from spiral galaxies is not from massive but from low-mass stars—in which case the equation above is invalid. As we discussed in §2.2 there is a long-running argument in the literature as to whether the bulk of the far-infrared emission from galaxies is from dust heated by old low-mass stars or from high-mass stars. In our models we have assumed the latter, but if the former is true this would at least partly explain the discrepancy, especially because the contribution of the high-mass stars, which will generally produce hotter dust, will be less in the submillimetre waveband. It would also, of course, invalidate the relationship between dust temperature and star-formation rate that we have assumed in our optically-thin models. However, this is probably not too important. In most of the disk models we have assumed a constant star-formation rate, so this would not be a concern, and in models in which the star-formation rate increases with redshift, although the relation may not be true at the lowest redshifts, it will eventually become true once the heating effect of the massive stars dominates the heating effect of the low-mass stars. For the elliptical models this should not be a concern, as the star-formation rates, and hence the heating effects of the massive stars, will be much greater than the star-formation rates in low-redshift spiral galaxies. It is also a potential problem for our two optically-thick disk-galaxy models, in which we assume that the submillimetre luminosity is proportional to the star-formation rate. However, in the model in which the star-formation rate is a constant this is not a problem, because the luminosity function at every redshift is exactly the same as the local luminosity function, which necessarily takes into account all the stars that are heating the dust. It is also probably not a major problem for the optically-thick model with the evolving star-formation rate, since although submillimetre luminosity may not be proportional to the star-formation rate at low redshift, it may well be true at high redshifts where the star-formation rate is higher.

Among the wide variety of source counts predicted by the different models (Figs 6-9) we can pick out one interesting qualitative feature. The $850 \mu\text{m}$ counts for the different

elliptical galaxy models all have the same shape, although they are offset horizontally and vertically from each other, except for the counts predicted using the one model in which we changed the shape of the low-redshift optical luminosity function. The reason for this is due to the characteristic spectral energy distribution of thermal dust emission. It is known (e.g. Hughes 1996) that if one is observing a galaxy at a wavelength well on the long-wavelength side of the peak of the thermal dust emission, the galaxy's flux will be approximately independent of redshift from $z \sim 1$ to the redshift at which emitted wavelength gets close to the wavelength of the peak. The reason for this is that at $z > 1$ the effect of increasing luminosity-distance on the flux is almost exactly cancelled by the fact that the emitted wavelength gets closer to the wavelength of the thermal peak, until the emitted wavelength is at the peak, at which redshift the two effects start to act in the same direction. At an observing wavelength of $850 \mu\text{m}$, the high temperature expected for the dust in spheroids in their formation phase means that this cancellation occurs from $1 \leq z \leq 10$. Thus, in this redshift range, which is where most of the sources are, there is a one-to-one mapping between submillimetre flux and luminosity, and the shape of the source counts will be the same as the shape of the distribution of submillimetre luminosities. Since the submillimetre luminosity at long wavelength is much more sensitive to dust mass than dust temperature (§2.3.2), this shape will also be the shape of the distribution of dust masses, which in turn will be approximately the same as the shape of the distribution of total masses; hence the sensitivity of the shape of the counts to the shape of the input luminosity function. Thus the shape of the counts is mainly governed by the distribution of masses of the systems that are doing the radiating, whereas the position of the counts in the diagram is governed by a combination of all the other factors that go into the models.

Finally, we look to the future. SCUBA and ISO should solve many of the problems with the disk-galaxy models: the reliance on a far-infrared luminosity function, rather than on a direct measurement of the submillimetre luminosity function; the gross simplification that all local galaxies have the same ratio of stellar mass to gas mass and the same far-infrared—submillimetre luminosity function. One additional limitation of our models is that we have made no attempt to incorporate the effects of different types of structure evolution, a choice we made partly because there is no consensus about how it occurs and partly because there is already sufficient variety in the predictions when we include more well-understood types of evolution. However, in principle there is no difficulty adapting the models to include it, and one can already use arguments based on the present models to show that including hierarchical clustering should have a relatively small effect on the background[†]. Finally SCUBA and ISO will allow us to test both the fundamental physics on which the models are based—once dust has been produced, does it stay within the

[†] In the hierarchical clustering scenario, one of the high-redshift galaxies in our models will actually be divided into a number of smaller units. However, the same mass of dust and same number of stars will be present. If we consider the optically-thin case first, the intensity of the interstellar radiation field in these smaller units may be somewhat different because of the different geometry, but the dust temperature depends only weakly on the intensity of the interstellar radiation field, and thus the total contribution to the background will be quite similar to what it is in the case of a single large galaxy. In the optically-thick case, all the starlight is assumed to be absorbed by dust, no matter the size

galaxy?—and the models directly by producing observed source counts with which to confront the predictions.

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of the structure in which the dust and stars reside, and so the predicted background will not change.

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Figure Captions

Fig. 1: Elementary models for the evolution of the mass of dust in a galaxy. The ordinate R is the ratio of mass of interstellar dust at gas fraction f to the interstellar dust mass at $f = 0.1$. The continuous curve is for the simple closed-box model. The dashed curves are for outflow models, with the lowest one having $\lambda/\alpha = 1.0$, the middle one having $\lambda/\alpha = 3.0$, and the highest one having $\lambda/\alpha = 10.0$. The dot-dashed curves are for inflow models with the lower having $\gamma/\alpha = 0.99$, and the higher having $\gamma/\alpha = 0.9$.

Fig. 2: A synthesis of the submillimetre observations that exist for nearby galaxies. For constructing a spectral energy distribution between the far-infrared and the submillimetre wavebands the most useful measurements that exist are the COBE spectrum of our galaxy (Wright et al. 1991) and the submillimetre measurements of other galaxies by Eales, Wynn-Williams & Duncan (1989) and by Stark et al. (1989), more recent measurements being made through a smaller aperture which exacerbates the problem of matching the submillimetre flux measurements to the IRAS far-infrared measurements. Despite the large aperture used by Eales et al., there is a problem correcting the IRAS measurements to the same aperture (Eales et al. 1989), whereas this is not a problem for the Stark et al. measurements because they measured integrated fluxes. For this reason we used the Stark et al. data to investigate the shape of the spectral energy distribution at wavelengths less than $360\mu\text{m}$, their longest-wavelength measurement, using the Eales et al. data at longer wavelengths. The figure shows the Stark et al. fluxes normalized so that each galaxy has a flux of 100 Jy at $100\mu\text{m}$, with the Eales et al. fluxes at $\lambda \geq 350\mu\text{m}$ normalized to a $350\mu\text{m}$ flux of 22.5 Jy, a figure chosen so that the shorter wavelength measurements would approximately agree with the Stark et al. data at $\lambda \sim 350\mu\text{m}$. Each galaxy is represented by a different symbol, and the spread of the symbols at the different wavelengths is an indication of the variation in galaxy SED's. The COBE spectrum of the galaxy has also been plotted on the diagram (thick line) and has been normalized to give as good an agreement as possible with the fluxes of the galaxies. The agreement between the SED's of the different galaxies is remarkably good (especially if one considers that some of dispersion is caused by measurement errors), and thus, given the present data, it is reasonable to use a single SED in the cosmological modelling of the disk population. The thin line shows the analytic model that is a good fit to the data and is used in the modelling. It consists of thermal emission from two dust components: one at 27K (dash) and one at 135K (dot-dash), with the mass of cold dust being 10^4 times greater than the mass of hot dust. We have assumed the emissivity law suggested by Hildebrand (1983), in which $\kappa_d \propto \nu^2$ at $\lambda > 250\mu\text{m}$ and $\kappa_d \propto \nu$ at $\lambda < 250\mu\text{m}$.

Figure 3: Predicted backgrounds from disk galaxies. The thick dot-dash line shows the background detected by Puget et al. (1996), the upper limits are the upper limits on the background determined by Davies et al. (1996). The thin lines show the predictions of various models. In all of the models disk galaxies are assumed to form at $z = 3$. Unless otherwise stated, it is assumed that $\Omega_0 = 1$ and that the star-formation rate is a constant. Lines A to F are the predictions of the optically-thin models. Line A (solid line) is the prediction of the closed-box model. Lines B-D (dashed) are the predictions for inflow and

outflow models (B—inflow, $\gamma/\alpha = 0.99$; C—outflow, $\lambda/\alpha = 1.0$; D—outflow, $\lambda/\alpha = 10.0$). Line E (dot-dash) is for a closed-box model again, but with the star-formation history derived by Pei and Fall (1995; see text). Line F (dots) is for a closed-box model, but with $\Omega_0 = 0$. Lines G and H (dot-dot-dot-dash) are for the optically-thick models, line G being the prediction for the model in which the star-formation rate is a constant, and line H for the model with the Pei and Fall star-formation history.

Figure 4: Predicted backgrounds from disk galaxies. The thick dot-dash line shows the background detected by Puget et al. (1996), the upper limits are the upper limits on the background determined by Davies et al. (1996). The thin lines show the predictions of various models. In all of the models disk galaxies are assumed to form at $z = 1$. Unless otherwise stated, it is assumed that $\Omega_0 = 1$ and that the star-formation rate is a constant. Lines A to F are the predictions of the optically-thin models. Line A (solid line) is the prediction of the closed-box model. Lines B-D (dashed) are the predictions for inflow and outflow models (B—inflow, $\gamma/\alpha = 0.99$; C—outflow, $\lambda/\alpha = 1.0$; D—outflow, $\lambda/\alpha = 10.0$). Line E (dot-dash) is for a closed-box model again, but with the star-formation history derived by Pei and Fall (1995; see text). Line F (dots) is for a closed-box model, but with $\Omega_0 = 0$. Lines G and H (dot-dot-dot-dash) are for the optically-thick models, line G being the prediction for the model in which the star-formation rate is a constant, and line H for the model with the Pei and Fall star-formation history.

Figure 5: Predicted backgrounds from elliptical galaxies. The thick dot-dash line shows the background detected by Puget et al. (1996), the upper limits are the upper limits on the background determined by Davies et al. (1996). The thin lines show the predictions of various models. In all of the models but one we assume that an individual galaxy forms in 0.1 Gyr, and in all the models but one we assume that $\Omega_0 = 1$. The solid lines are for a closed-box model. Line A is for a model in which the galaxies form in the redshift interval $5 < z < 10$; line B is for a model in which the galaxies form in the redshift range $2 < z < 5$ with an individual galaxy forming in 1 Gyr; line C is for a model in which the galaxies form in the redshift range $2 < z < 5$ with an individual galaxy forming in 0.1 Gyr. The dotted line (D) is for an inflow model ($\gamma/\alpha = 0.9$) in which galaxies form in the redshift range $5 < z < 10$. The dot-dash line (E) is for a closed-box model again, in which the galaxies form in the redshift range $5 < z < 10$ but in which $\Omega_0 = 0$. The dashed line (F) is for an outflow model ($\lambda/\alpha = 4.0$) in which galaxies form in the redshift range $5 < z < 10$.

Figure 6: Differential source counts at $190 \mu\text{m}$ for disk galaxy models. The ordinate is number arcmin^{-2} . In all of the models disk galaxies are assumed to form at $z = 3$. Unless otherwise stated, we assume $\Omega_0 = 0$ and that the star-formation rate does not change with time. The symbols are as follows: optically-thin closed-box—circles with dots in centres; optically-thin inflow ($\gamma/\alpha = 0.99$)—dots; optically-thin outflow ($\lambda/\alpha = 1.0$)—circles; optically-thin outflow ($\lambda/\alpha = 10.0$)—crosses; optically-thin closed-box with star-formation history from Pei & Fall (1995, see text)—squares; optically-thin closed-box with $\Omega_0 = 0$ —triangles; optically-thick—crosses in circles; optically-thick with Pei and Fall star-formation history—asterisks.

Figure 7: Differential source counts at $850\ \mu\text{m}$ for disk galaxy models. The ordinate is number arcmin^{-2} . In all of the models disk galaxies are assumed to form at $z = 3$. Unless otherwise stated, we assume $\Omega_0 = 0$ and that the star-formation rate does not change with time. The symbols are as follows: optically-thin closed-box—circles with dots in centres; optically-thin inflow ($\gamma/\alpha = 0.99$)—dots; optically-thin outflow ($\lambda/\alpha = 1.0$)—circles; optically-thin outflow ($\lambda/\alpha = 10.0$)—crosses; optically-thin closed-box with star-formation history from Pei & Fall (1995, see text)—squares; optically-thin closed-box with $\Omega_0 = 0$ —triangles; optically-thick—crosses in circles; optically-thick with Pei and Fall star-formation history—asterisks.

Figure 8: Differential source counts at $190\ \mu\text{m}$ for elliptical galaxy models. The ordinate is number arcmin^{-2} . In all the models but one we assume that $\Omega_0 = 1$ and we assume a closed-box model for dust evolution unless otherwise stated. In all the models but one we have assumed the optical luminosity function mentioned in the text. In the one exception we have investigated the effect of changing the luminosity function, by changing the value of the Schechter α parameter to -1.0, while leaving the other Schechter parameters the same. In this model, for which the predictions are shown by open circles with crosses inside, we have assumed that galaxies form in the redshift range $5 < z_f < 10$ and that an individual galaxy forms in a period (τ) of 0.1 Gyr. The symbols for the other models are as follows: $2 < z_f < 5$, $\tau = 1$ Gyr—dots in circles; $2 < z_f < 5$, $\tau = 0.1$ Gyr—dots; $5 < z_f < 10$, $\tau = 0.1$ Gyr—open circles; outflow ($\lambda/\alpha = 4.0$), $5 < z_f < 10$, $\tau = 0.1$ Gyr—diagonal crosses; inflow ($\gamma/\alpha = 0.9$), $5 < z_f < 10$, $\tau = 0.1$ Gyr—squares; $\Omega_0 = 0$, $5 < z_f < 10$, $\tau = 0.1$ Gyr—triangles.

Figure 9: Differential source counts at $850\ \mu\text{m}$ for elliptical galaxy models. The ordinate is number arcmin^{-2} . In all the models but one we assume that $\Omega_0 = 1$ and we assume a closed-box model for dust evolution unless otherwise stated. In all the models but one we have assumed the optical luminosity function mentioned in the text. In the one exception we have investigated the effect of changing the luminosity function, by changing the value of the Schechter α parameter to -1.0, while leaving the other Schechter parameters the same. In this model, for which the predictions are shown by open circles with crosses inside, we have assumed that galaxies form in the redshift range $5 < z_f < 10$ and that an individual galaxy forms in a period (τ) of 0.1 Gyr. The symbols for the other models are as follows: $2 < z_f < 5$, $\tau = 1$ Gyr—dots in circles; $2 < z_f < 5$, $\tau = 0.1$ Gyr—dots; $5 < z_f < 10$, $\tau = 0.1$ Gyr—open circles; outflow ($\lambda/\alpha = 4.0$), $5 < z_f < 10$, $\tau = 0.1$ Gyr—diagonal crosses; inflow ($\gamma/\alpha = 0.9$), $5 < z_f < 10$, $\tau = 0.1$ Gyr—squares; $\Omega_0 = 0$, $5 < z_f < 10$, $\tau = 0.1$ Gyr—triangles.

















